PARABOLAS

Basic Skills

1. Find the values of p for which the equation $x^2 - px + p + 3 = 0$ has equal roots.

   (4)

2. For what values of c does the equation $c(x^2 + 1) = 5x$ have equal roots ?

   (4)

3. Find b, given that the equation $(bx)^2 + (2 - 2b)x + 4 = 0$ has equal roots.

   (4)

4. Find w if the equation $(4w - 1)x^2 + (2w + 1)x + 1 = 0$ has equal roots.
   Find the nature of the roots when w lies between these values.

   (3,2)

5. Determine the range of values of v for which the equation
   $x^2 + x(3 - v) - v + 6 = 0$ has real roots.

   (5)

6. Show that the roots of the equation $x^2 + 6x + 11 = n(2x + 7)$
   are unreal if $-2 < n < 1$.

   (5)

7. Prove that the equation $y^2 + (t + 2)y + (2t + 1) = 0$ has real roots
   when $t \leq 0$ or $t \geq 4$.

   (5)

8. Prove that the equation $(m + 2)x^2 - (3m + 2)x + 2m = 0$ has real roots
   for all values of m, $m \in R$.

   (5)

9. Prove that the roots of the equation $(2b - 3)x^2 + 2(b + 1)x + 1 = 0$
   are real provided b is real.

   (5)

10. Show that the roots of $t(x + 4)(x + 1) = 9x$ are unreal if $1 < t < 9$.

    (5)

11. Show that the line $y = 4x + 3$ is a tangent to the parabola $f(x) = x^2 + 7$.

(3)

12. For what value of c is the line $y = x + c$ a tangent to the parabola
$y = 2x^2 + 5x + 3$?

(5)

13. For what value(s) of m is the line $y = mx + 1$ a tangent to the parabola
$g(x) = x^2 + x + 5$.

(5)

14. Express $x^2 + 7x + 25$ in the form $(x + b)^2 + c$.
State the minimum value of $x^2 + 7x + 25$ and justify your answer.

(3)

15. By expressing $k(x) = 12 - x - x^2$ in the form $a - (x - b)^2$ determine
the maximum value of k.

(3)

16. Express $x^2 - 5x + 9$ in the form $(x + a)^2 + b$.

State the maximum value of $\dfrac{2}{x^2 - 5x + 9}$

(3)

17. Prove that $6x^2 - 3x + 1 \geq 0.625$ for all real values of x.

(3)

18. Obtain the equation of the parabola which has roots -2 and 5 and which passes
through the point $(2,30)$.

(3)

19. Find the equation of the parabola which passes through the origin and has a turning
point at $(-3,-18)$.

(4)

20. Find the equation of the parabola which has a minimum turning point at $(-2,0)$
and which cuts the y-axis at $(0,2)$.

(3)

21. The parabola $f(x)$ has a turning point at $(3,5)$ and cuts the y-axis at $(0,12)$.
Sketch the parabolas $y = f(x)$, $y = f(x) - 5$ and $y = f(x + 2)$

(5)

22. A parabola passes through the points $(0,8)$, $(2,-1)$ and $(4,-1)$. Find the
equation of the parabola.

(4)

PARABOLAS   Basic skills   Answers

1. $p = 6$ or -2

2. $c = 2.5$ or -2.5

3. $b = \frac{1}{3}$ or -1

4. $w = \frac{1}{2}$ or $\frac{5}{2}$; $\frac{1}{2} < w < \frac{5}{2}$, No roots

5. $v < -3$ or $v > 5$

6.   Show $b^2 - 4ac < 0$

7.   Show $b^2 - 4ac \geq 0$

8.   Show $b^2 - 4ac \geq 0$

9.   Show $b^2 - 4ac \geq 0$

10.   Show $b^2 - 4ac < 0$

11.   Equal roots $\therefore$ Tangent

12. $c = 1$

13. $m = -3$ or 5

14. $b = \frac{7}{2}$, $c = 12\frac{3}{4}$ Min T.P $(-\frac{7}{2} , 12\frac{3}{4})$

15.   Max T.P $(\frac{1}{2} , 12\frac{1}{4})$

16. $a = -\frac{5}{2}$, $b = 2\frac{3}{4}$, Max value $= \frac{8}{11}$

17.   Show $b^2 - 4ac < 0$

18. $y = -\frac{5}{2}(x + 2)(x - 5)$

19. $y = -x(x - 6)$

20. $y = \frac{1}{2}x^2 + 2x + 2$

21.   Sketch the graphs

22. $y = \frac{9}{8}x^2 - \frac{27}{4}x + 8$